Ostatnim schronieniem ludzi, którym nie chciało się wojować, był garnizon. Znałem pewnego suplenta, który jako matematyk nie chciał strzelać z armat i ukradł jakiemuś porucznikowi zegarek, żeby się tylko dostać do garnizonu. Zrobił to po gruntownym namyśle. Wojna mu nie imponowała i nie zachwycała go. Strzelanie do nieprzyjaciela i zabijanie takich samych nieszczęśliwych suplentów-matematyków po stronie przeciwnejuważał za idiotyzm.

— Nie chcę być znienawidzony za swoje czyny — powiedział sobie i z premedytacją ukradł zegarek.

Najpierw badali jego stan umysłowy, ale gdy oświadczył, że chciał się zbogacić, wyprawili go do garnizonu.

Czasem to co niezmiennicze bywa nieźle zakręcone

Michał Wojciechowski

Tiling with Polyominoes and Combinatorial Group Theory

J. H. CONWAY

Princeton University, Princeton, New Jersey

AND

J. C. LAGARIAS

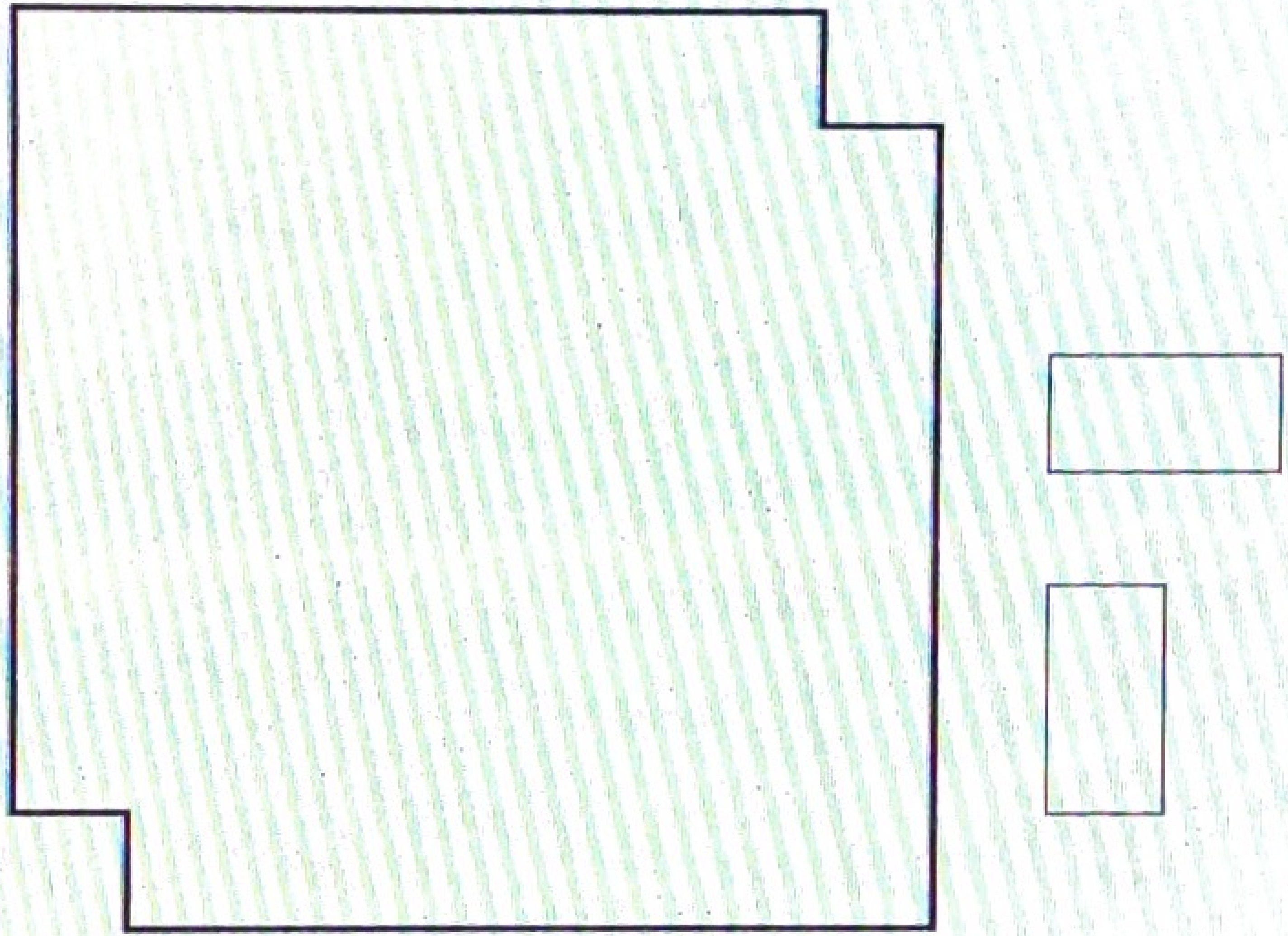
AT&T Bell Laboratories, Murray Hill, New Jersey

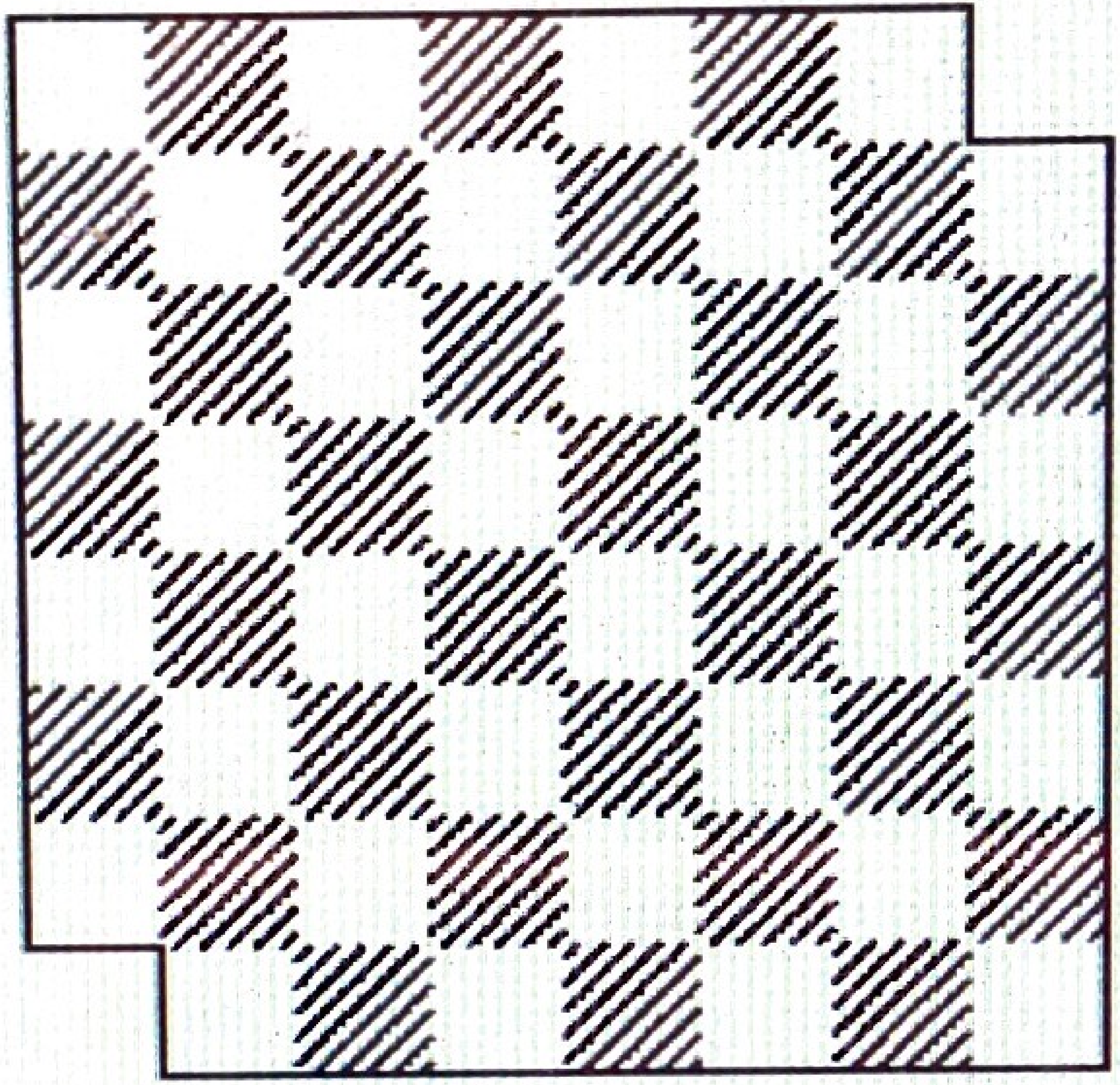
Communicated by Andrew Odlyzko

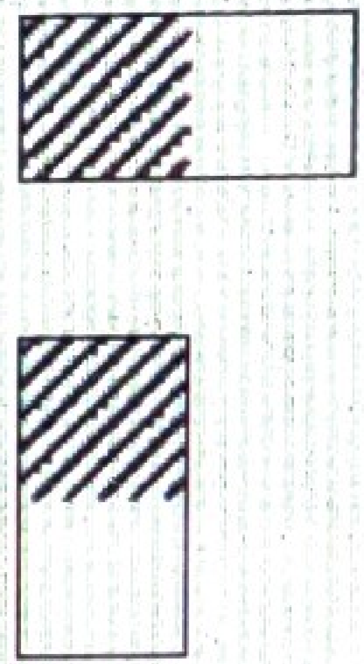
Received May 3, 1988

When can a given finite region consisting of cells in a regular lattice (triangular, square, or hexagonal) in $\mathbb{R}^2$ be perfectly tiled by tiles drawn from a finite set of tile shapes? This paper gives necessary conditions for the existence of such tilings using *boundary invariants*, which are combinatorial group-theoretic invariants associated to the boundaries of the tile shapes and the regions to be tiled. Boundary invariants are used to solve problems concerning the tiling of triangular-shaped regions of hexagons in the hexagonal lattice with certain tiles consisting of three hexagons. Boundary invariants give stronger conditions for nonexistence of tilings than those obtainable by weighting or coloring arguments. This is shown by considering whether or not a region has a *signed tiling*, which is a placement of tiles assigned weights 1 or -1, such that all cells in the region are covered with total weight 1 and all cells outside with total weight 0. Any coloring (or weighting) argument that proves nonexistence of a tiling of a region also proves nonexistence of any signed tiling of the region as well. A partial converse holds: if a simply connected region has no signed tiling by simply connected tiles, then there is a generalized coloring argument proving that no signed tiling exists. There exist regions possessing a signed tiling which can be shown to have no perfect tiling using boundary invariants.

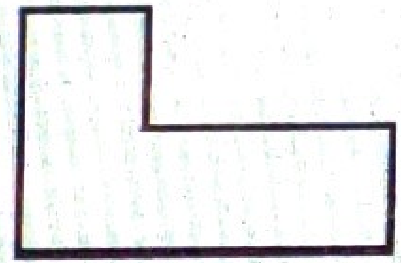

15·92

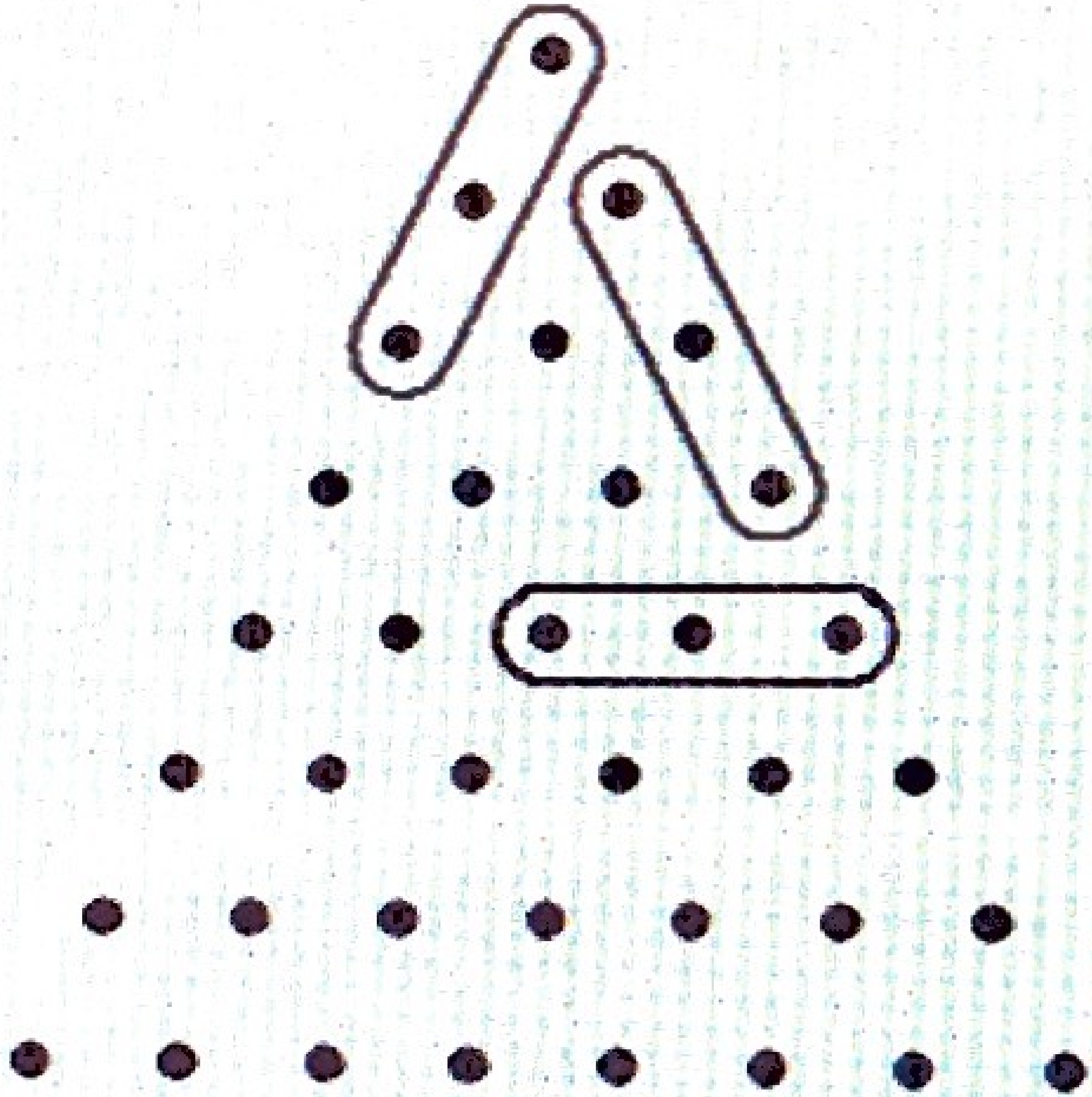






1	1	1	1	1	1	1	1	1	1
5	5	5	5	5	5	5	5	5	5
1	1	1	1	1	1	1	1	1	1
5	5	5	5	5	5	5	5	5	5
1	1	1	1	1	1	1	1	1	1
5	5	5	5	5	5	5	5	5	5
1	1	1	1	1	1	1	1	1	1
5	5	5	5	5	5	5	5	5	5
1	1	1	1	1	1	1	1	1	1
5	5	5	5	5	5	5	5	5	5





0

1 1

2 2 2

0 0 0 0

1 1 1 1 1

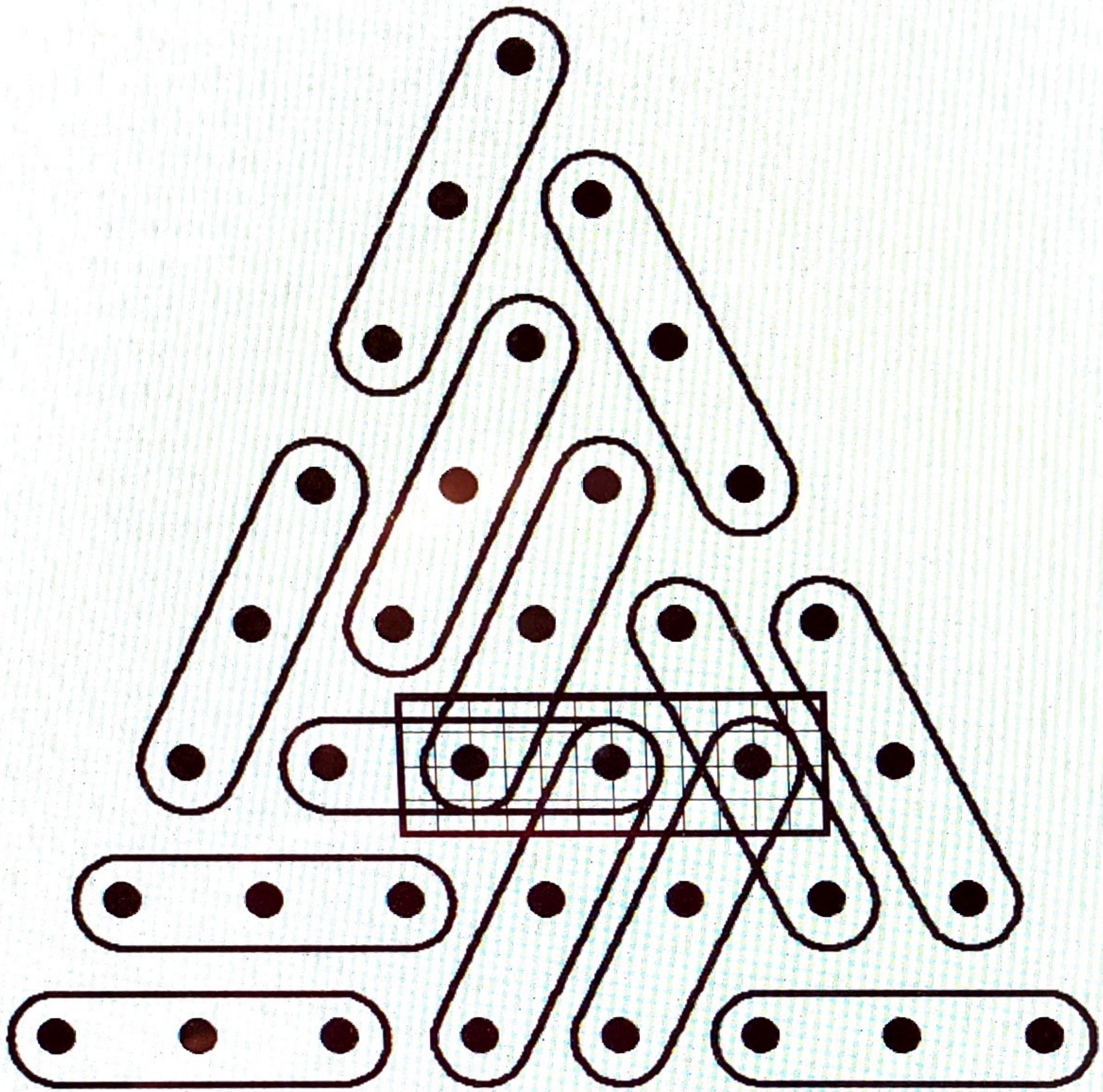
2 2 2 2 2 2

0 0 0 0 0 0 0

1 1 1 1 1 1 1 1

2 2 2 2 2 2 2 2 2

0 0 0 0 0 0 0 0 0 0



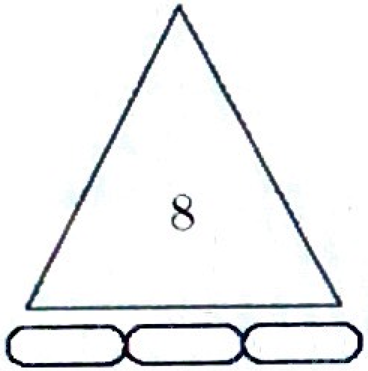
SUMA LICZB W TRÓJKĄCIE O BOKU n WYNOSI KOLEJNO MOD 3

0, 2, 2, 2, 1, 1, 1, 0, 0

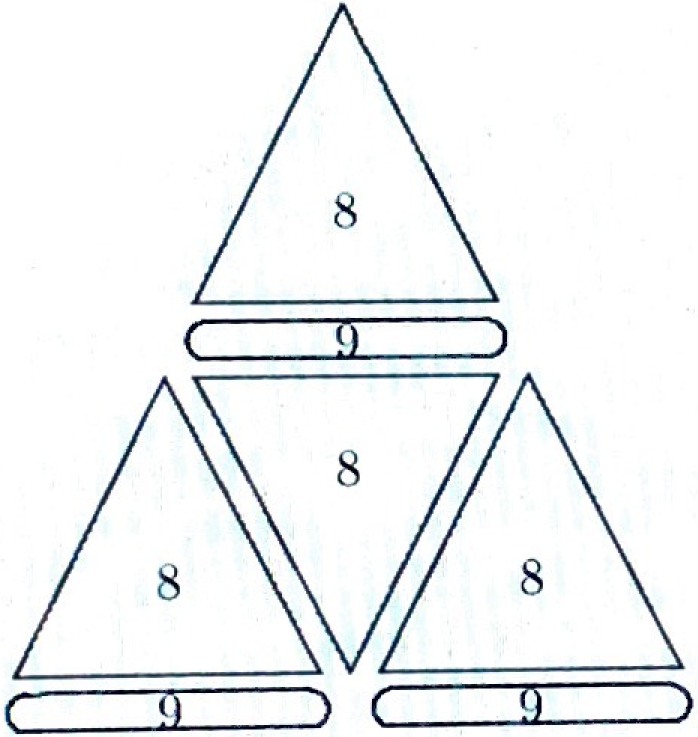
STĄD $n \equiv 1, 8, 0 \mod 9$

ALE $3 \mid \frac{n(n+1)}{2} \Rightarrow n \equiv 0, 2 \mod 3$

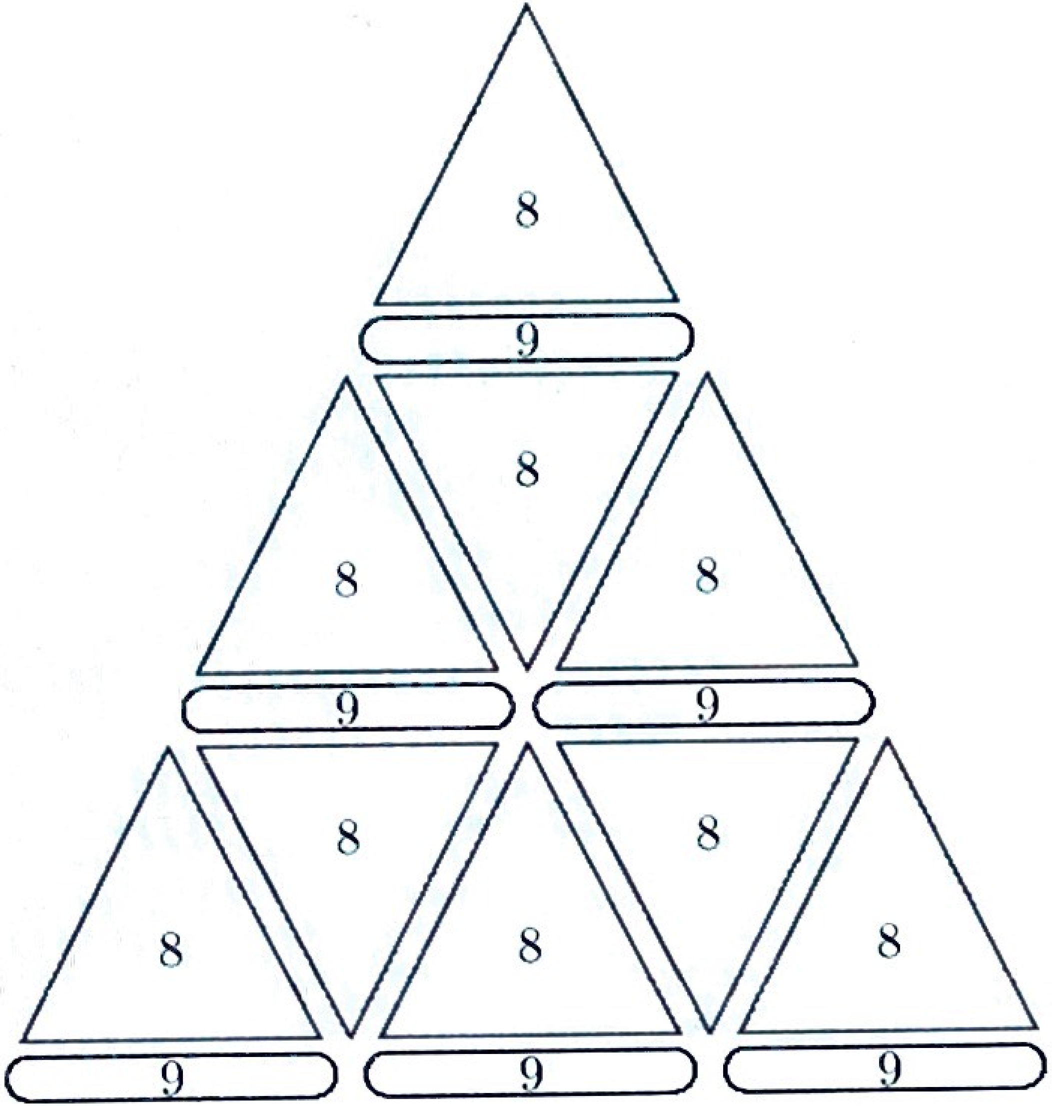
STĄD $n = 8, 0 \mod 9$

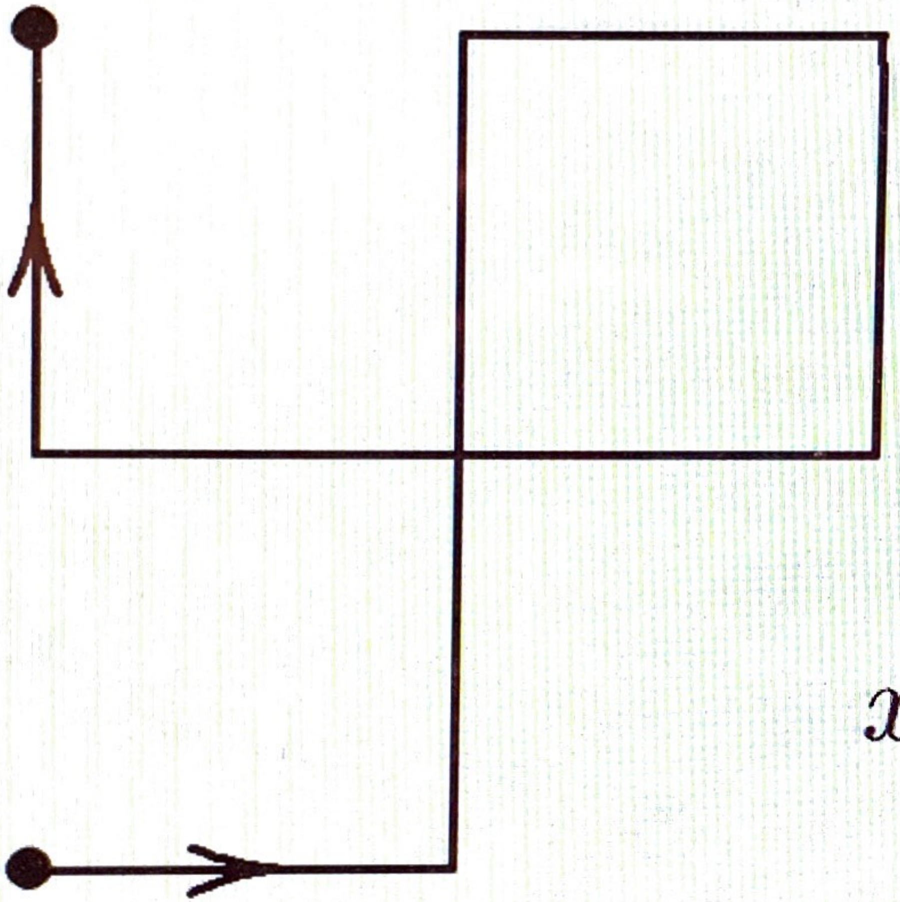

8

,

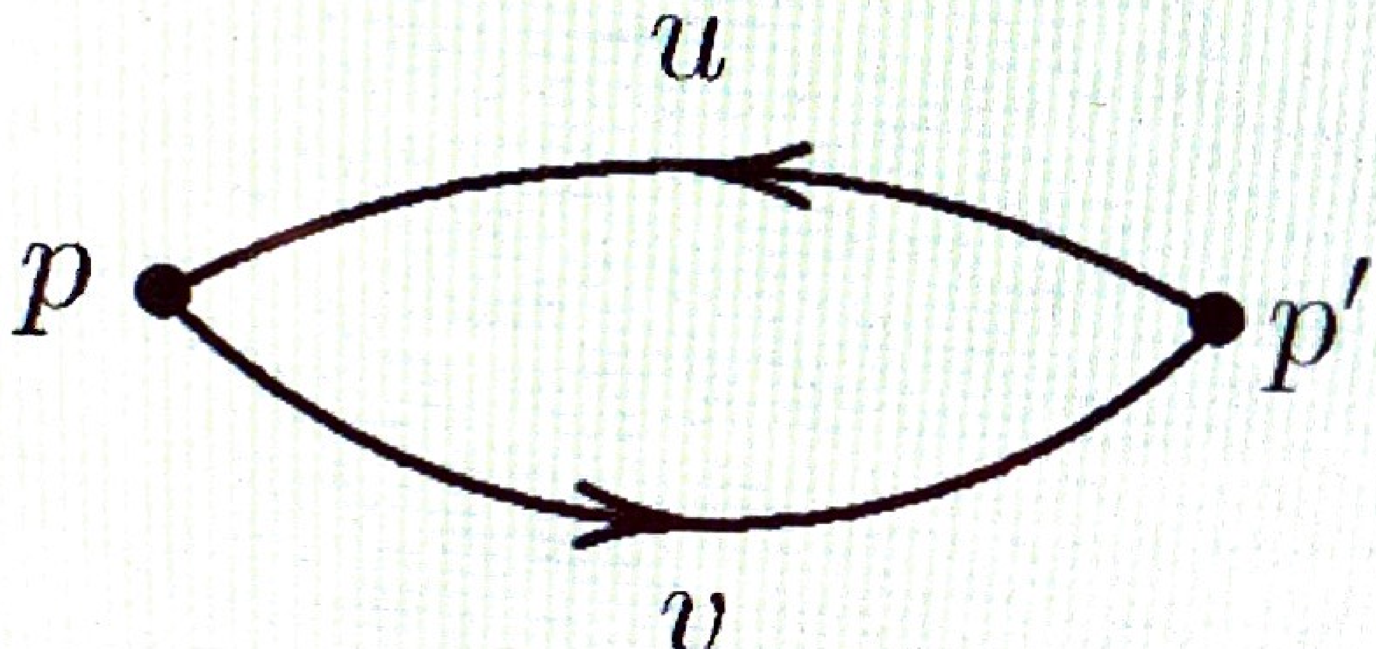

8
9
8
8
8
9
9

,


8
9
8
8
8
9
9
8
8
8
8
8
9
9
9



$xy^2xy^{-1}x^{-2}y$


u
p
p′
v

KODOWANIE NIE ZALEŻY OD PUNKTU STARTU NA BRZEGU

$w_i = uv \qquad w_i' = vu$

$uv = w_i = e$

$vu = (u^{-1}u)(vu) = u^{-1}(uv)u = u^{-1}u = e.$

$T_1, \ldots, T_n$ – figury

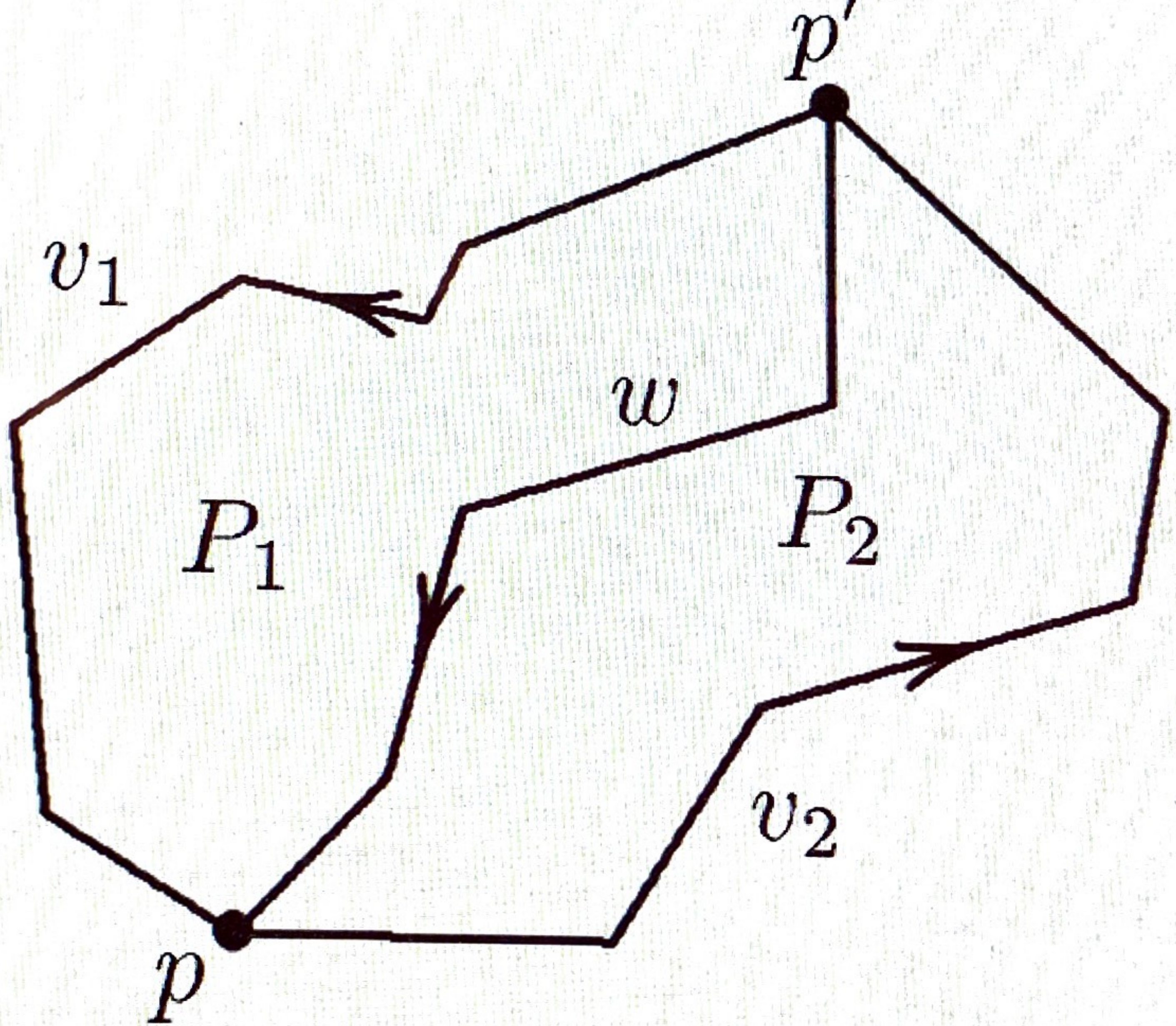
$w_1 \ldots w_n$ – słowa brzegowe

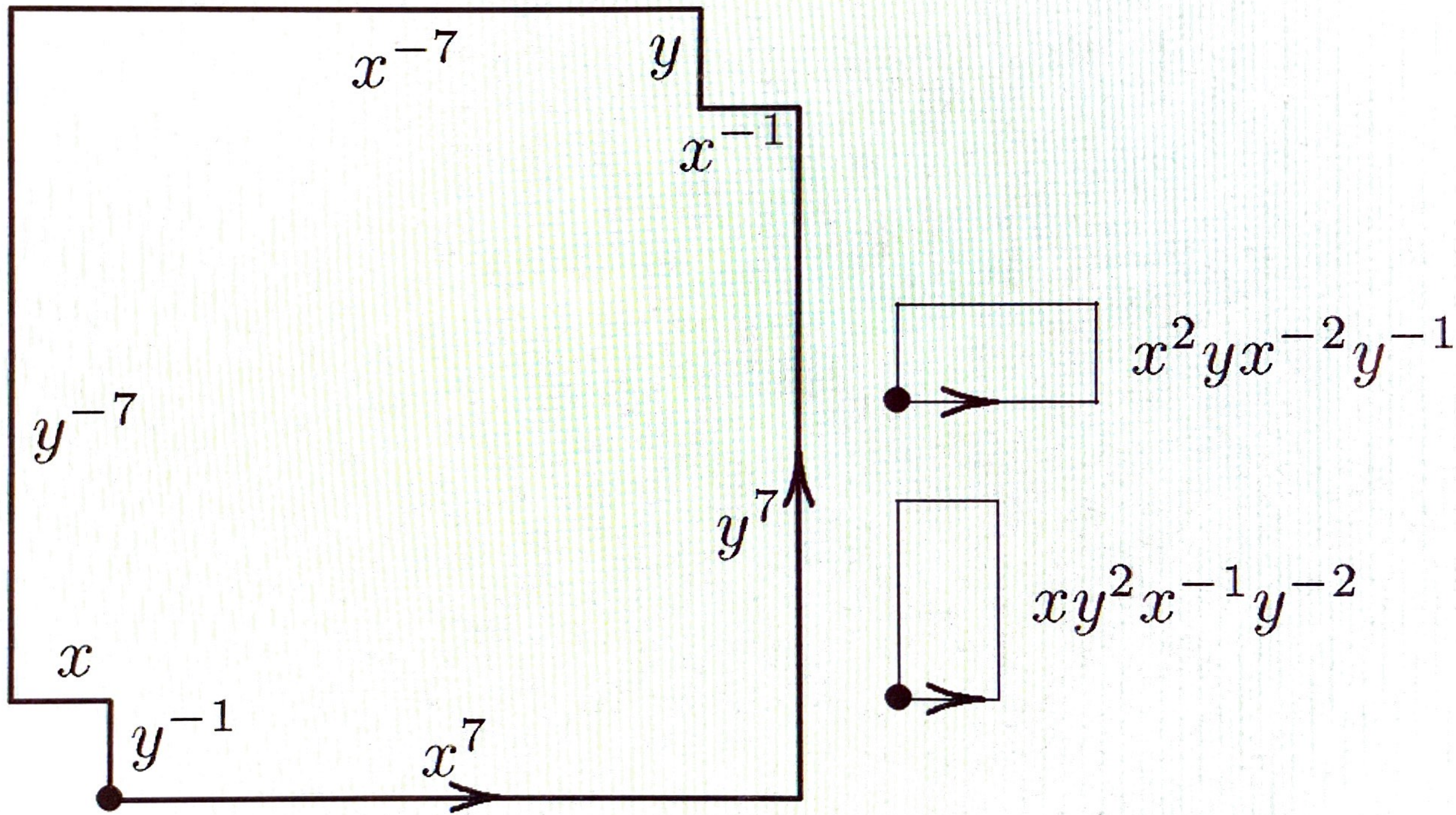
P – duża figura

U – jej słowo brzegowe

JEŚLI P JEST POKRYTE FIGURAMI $T_1, \ldots, T_n$ i $w_1 = e, w_2 = e, \ldots, w_n = e$ TO

$U = e$.


p'
v_1
w
P_1
P_2
v_2
p



x^{-7}
y
x^{-1}
y^{-7}
y^7
x
y^{-1}
x^7
$x^2yx^{-2}y^{-1}$
$xy^2x^{-1}y^{-2}$

$$w_1 = x^2 y x^{-2} y^{-1}$$

$$w_2 = x y^2 x^{-1} y^{-2}$$

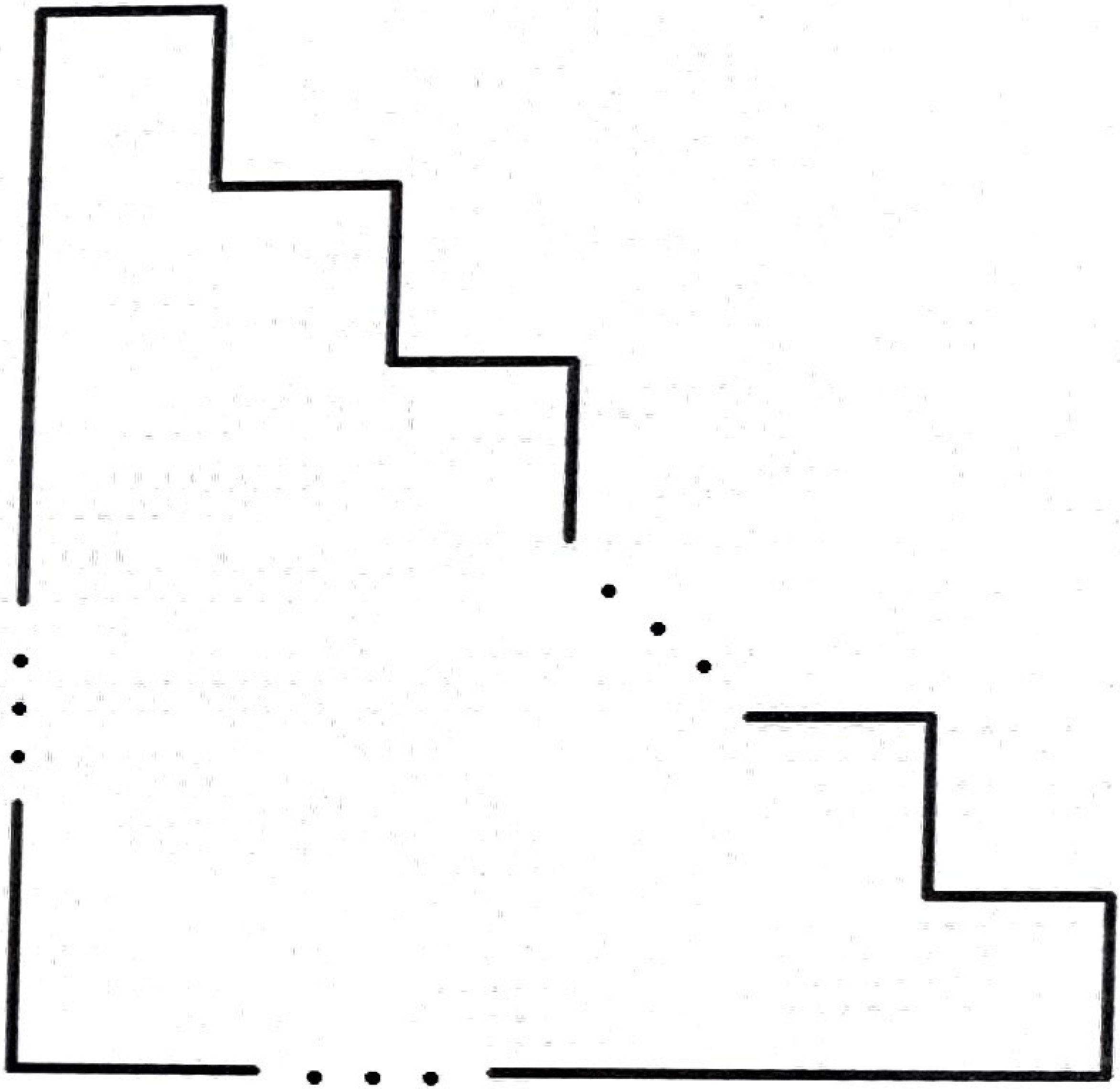
$$u = x^7 y^7 x^{-1} y x^{-7} y^{-7} x y^{-1}$$

$x = (2\,1\,3)$ $\quad x^2 = (1\,2\,3)$

$y = (1\,3\,2)$ $\quad y^2 = (1\,2\,3)$

$w_1 = e \quad , \quad w_2 = e$

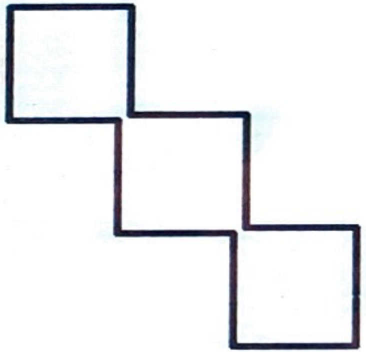
$u = (3\,1\,2)$



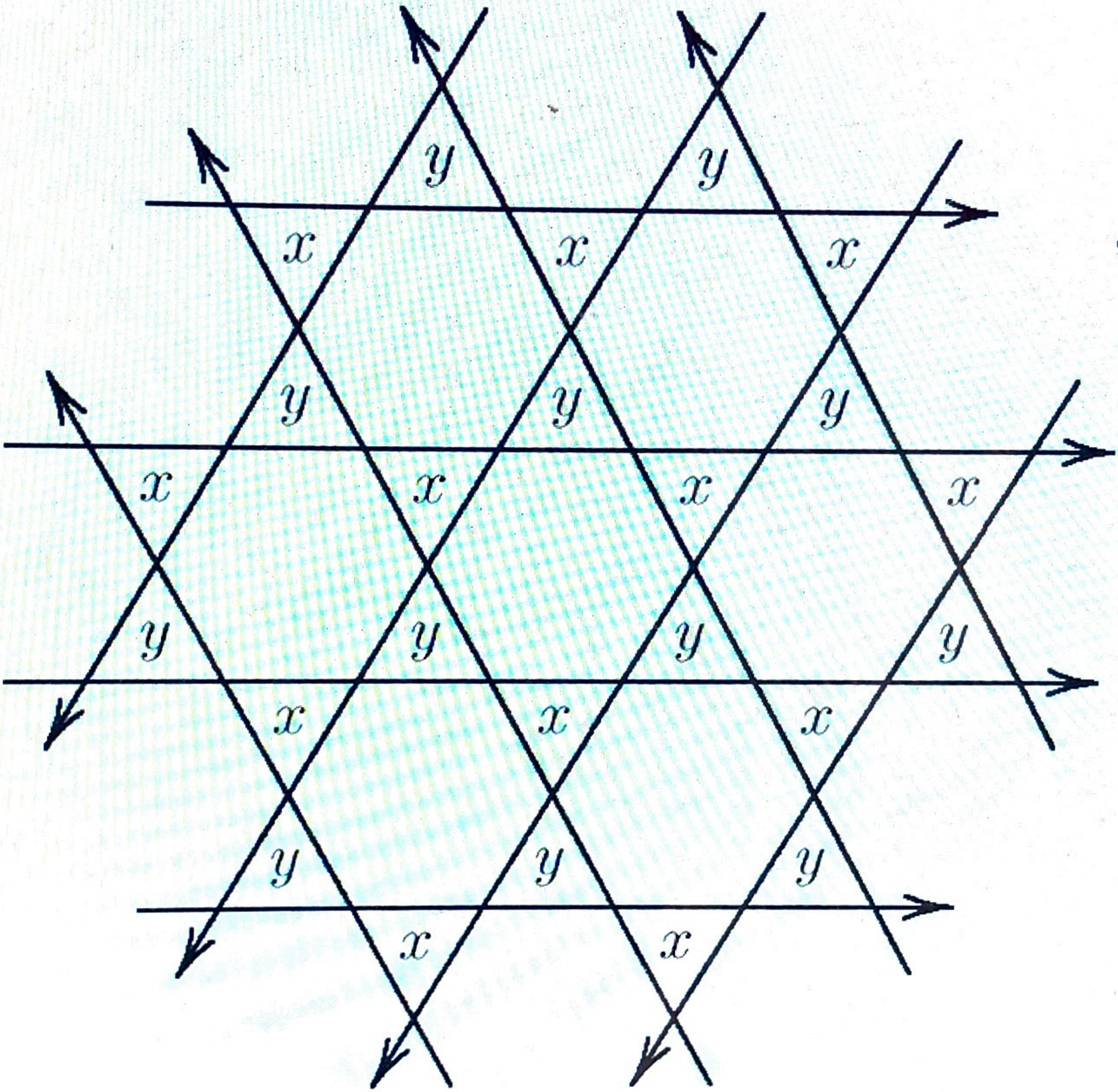
$$U_n = y^{-n} x^n (yx^{-1})^n$$

$$W_1 = x^3 y x^{-3} y^{-1}$$

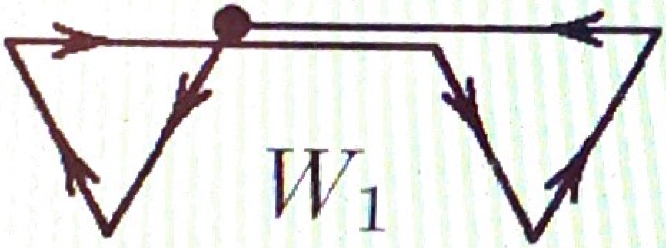
$$W_2 = x y^3 x^{-1} y^{-3}$$



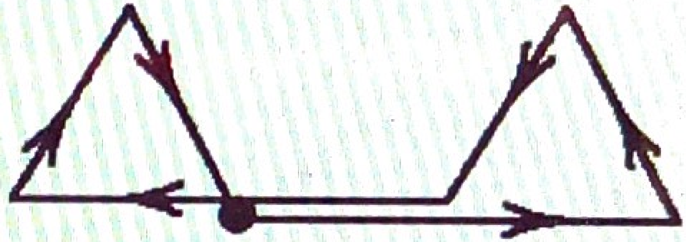
$$W_3 = (y x^{-1})^3 (y^{-1} x)^3$$



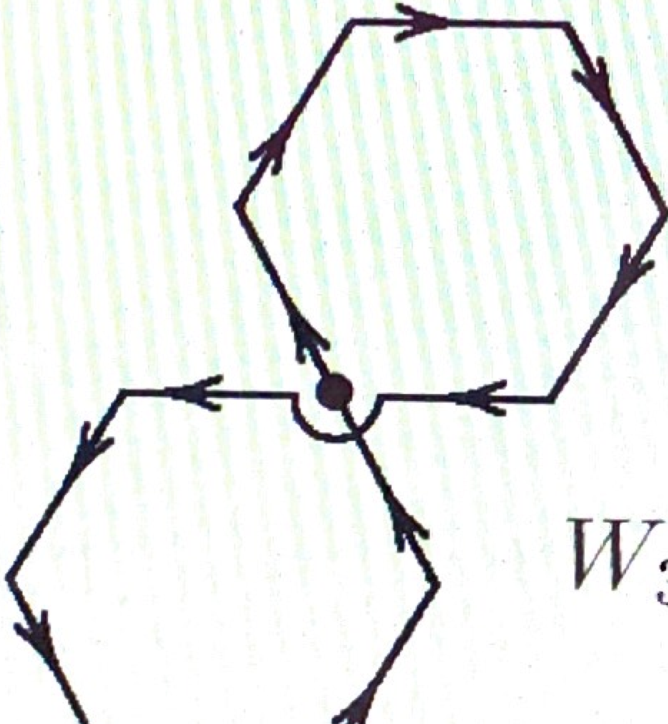
y
y
x
x
x
y
y
y
x
x
x
x
y
y
y
y
x
x
x
y
y
y
x
x



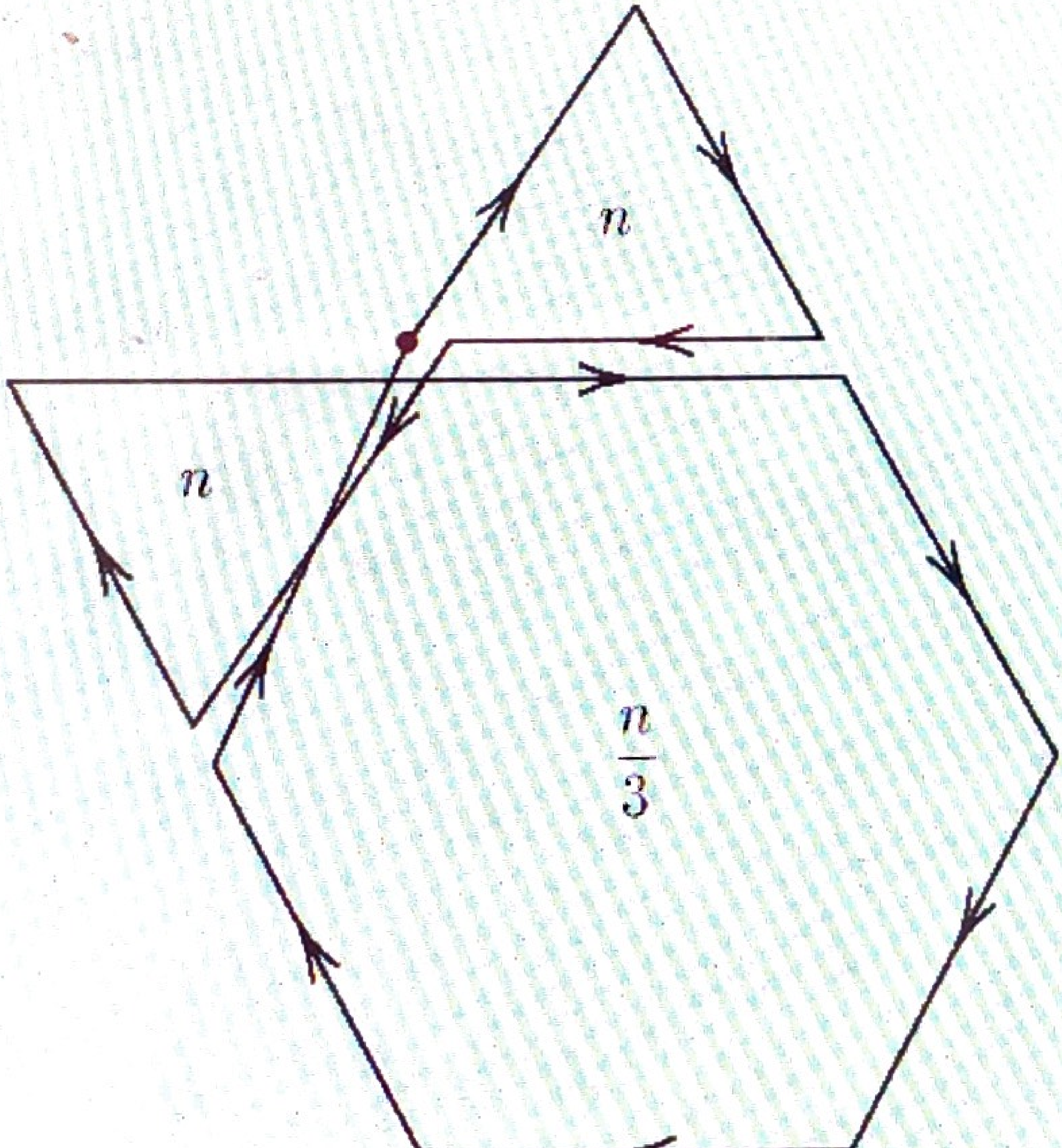
W_1

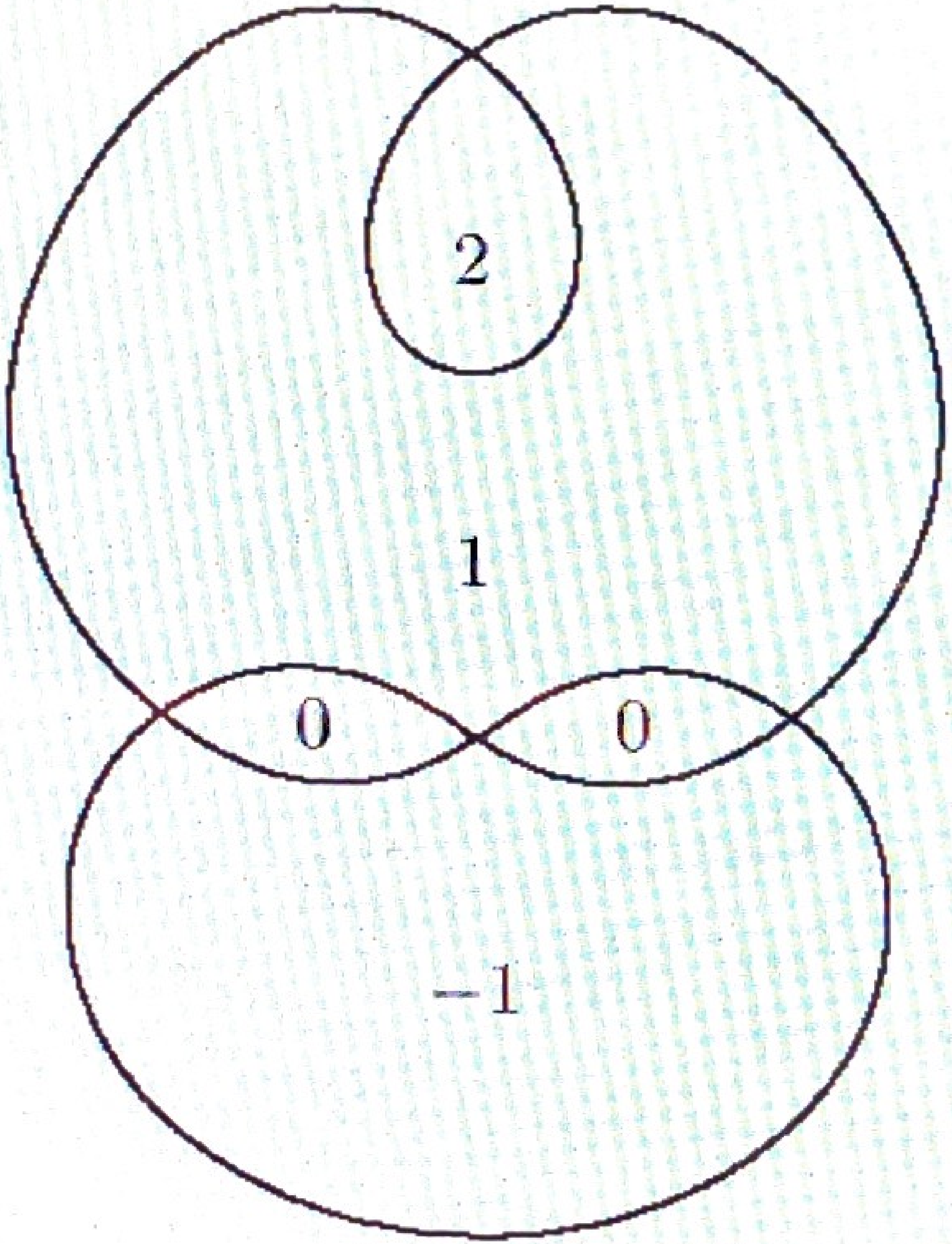


W_2



W_3


n
n
n/3


2
1
0
0
−1